VILNIUS TECH
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Expert Methods for Research in Social Sciences

Agenda

- Introduction to MCDM methods
- Application of SAW
- Application of TOPSIS
- Application of EDAS
- Kendall Coefficient of Concordance
- Experts’ Competence Coefficient

Multiple Criteria Decision-Making (MCDM) Methods

Selection of the best from a set of alternatives each of which is evaluated against multiple criteria.

Simple Additive Weighting (SAW)

The main concept of SAW

$$S_j = \sum_{i=1}^{m} w_i \, \bar{r}_{ij}$$

w_i – weight of the *i*-th criterion

$\bar{r}_{ij}$ – normalised *i*-th criterion's value for *j*-th object; $i = 1, \ldots, m; j = 1, \ldots, n$

m – the number of the criteria used

n – is the number of the objects (alternatives) compared.

Maximising vs Minimising Criteria

$$\bar{r}_{ij} = \frac{\min\limits_{j} r_{ij}}{r_{ij}}$$

r_{ij} – *i*-th criterion's value for *j*-th alternative

$\min\limits_{j} r_{ij}$ – the smallest *i*-th criterion's value for all the alternatives compared

$\bar{r}_{ij}$ – denotes the converted values.

Maximising vs Minimising Criteria

$$\bar{r}_{ij} = \frac{\bar{r}_{ij}}{\max_j r_{ij}}$$

$\max_j r_{ij}$ – the largest *i*-th criterion’s value of all alternatives

Transformation

$$\hat{r}_{ij} = r_{ij} + \left|\min_j r_{ij}\right| + 1$$

Limitations of SAW

- Maximisation
- Positive values

Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS)

Essentials of TOPSIS

- In this method two artificial alternatives are hypothesised:
  - **Ideal alternative**: the one which has the best level for all attributes considered.
  - **Negative ideal alternative**: the one which has the worst attribute values.
- TOPSIS selects the alternative that is the closest to the ideal solution and farthest from negative ideal alternative.
- TOPSIS assumes that we have ***m*** alternatives (options) and ***n*** attributes/criteria and we have the score of each option with respect to each criterion.

TOPSIS Steps

1. Construct the decision matrix and determine the weight of criteria.
2. Calculate the normalized decision matrix.
3. Calculate the weighted normalised decision matrix.
4. Determine the positive ideal and negative ideal solutions.
5. Calculate the separation measures from the positive ideal solution and the negative ideal solution.
6. Calculate the relative closeness to the positive ideal solution.
7. Rank the preference order or select the alternative closest to 1.

Step 1

Let $X = x_{ij}$ be a decision matrix, and let $W = [w_1, w_2, \ldots, w_n]$ significance vector (weight), where $x_{ij} \in \Re, w_j \in \Re$ ir $w_1 + w_2 + \cdots + w_n = 1$.

Step 2

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$$

Step 3

$$v_{ij} = w_j n_{ij} \text{ for } i = 1, \dots, m; j = 1, \dots, n.$$

Step 4

$$V^{+} = (v_1^{+}, v_2^{+}, \dots, v_n^{+}) = \left(\left(\max_i v_{ij} \middle| j \in I \right), \left(\min_i v_{ij} \middle| j \in J \right) \right)$$

$$V^{-} = (v_1^{-}, v_2^{-}, \dots, v_n^{-}) = \left(\left(\min_i v_{ij} \middle| j \in I \right), \left(\max_i v_{ij} \middle| j \in J \right) \right)$$

Step 5

$$S_i^+ = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^+\right)^2}, i = 1, 2, \dots, m$$

$$S_i^- = \sqrt{\sum_{j=1}^{n}\left(v_{ij} - v_j^-\right)^2}, i = 1, 2, \dots, m$$

Step 6

$$P_i = \frac{S_i^-}{S_i^- + S_i^+}, 0 \leq P_i \leq 1, i = 1, 2, \dots, m.$$

Step 7

Alternatives are ranked based on the decreasing value of P_i.

Evaluation Based on Distance from Average Solution (EDAS)

Step 1

$$X = [X_{ij}]_{m \times n} = \begin{bmatrix} X_{11} & \cdots & X_{1m} \\ \vdots & \ddots & \vdots \\ X_{n1} & \cdots & X_{nm} \end{bmatrix},$$

where:

X_{ij} – value of *i*-th alternative on *j*-th criterion

n – number of alternatives

m – number of criteria

Step 1

If there are negative values, the inital matrix should be transformed by using the following formula:

$$x'_{ij} = x_{ij} - \min_j x_{ij}.$$

Step 2

$$AV_j = \frac{\sum_{i=0}^{n} X_{ij}}{n},$$

where:

AV_j – the average solution.

Step 3

if *j*-th criterion is beneficial:

$$PDA_{ij} = \frac{\max(0,(W_{ij} - AV_j))}{AV_j},$$

$$NDA_{ij} = \frac{\max(0,(AV_j - X_{ij}))}{AV_j},$$

if *j*-th criterion is cost:

$$PDA_{ij} = \frac{\max(0,(AV_j - X_{ij}))}{AV_j},$$

$$NDA_{ij} = \frac{\max(0,(X_{ij} - AV_j))}{AV_j},$$

where:

PDA_{ij} – positive distance from average

NDA_{ij} – negative distance from average

Step 4

$$NSP_i = \frac{\sum_{j=1}^{m} w_j PDA_{ij}}{max_i(SP_i)},$$

$$NSN_i = 1 - \frac{\sum_{j=1}^{m} w_j NDA_{ij},}{max_i(SN_i)},$$

where:

NSP_i – normalised value of weighted sum of PDA_{ij}

NSN_i – normalised value of weighted sum of NDA_{ij}

Step 5

$$AS_i = \frac{1}{2}(NSP_i + NSN_i),$$

where:

AS_i – appraisal score, $0 \le AS_i \le 1$.

Step 6

The last step includes alternatives' ranking according to the decreasing values of AS_i.

Kendall‘s Coefficient of Concordance

Kendall‘s W

- **Kendall’s coefficient of concordance** (aka **Kendall’s W**) is a measure of the agreement among several quantitative or semiquantitative variables that are assessing a set of objects of interest.

- Proposed by Maurice G. Kendall and Bernard Babington Smith.

Hypotheses

H_0: There is no agreement between the judges ($W = 0$).

H_a: There is an agreement between the judges ($W \neq 0$).

Kendall‘s W Calculation

$$a = 0{,}5m(k+1)$$

$$S^2 = \sum_{j=1}^{k}\left(\sum_{j=1}^{m} x_{ij} - a\right)^2$$

$$T_l = \sum_{q=1}^{u}(t_q^3 - t_q)$$

$$W = \frac{12S^2}{m^2(k^3-k)-mT}$$

$$\chi^2 = Wm(k-1)$$

Experts' Competence

1. $K_i^0 = \frac{1}{m}$
2. $x_j^t = \sum_{i=1}^{m} K_i^{t-1} \cdot x_{ij}$
3. $\lambda^t = \sum_{j=1}^{n} \sum_{i=1}^{m} x_j^t \cdot x_{ij}$
4. $K_i^t = \frac{1}{\lambda^t} \sum_{j=1}^{n} x_j^t \cdot x_{ij}, \sum_{i=1}^{m} K_i^t = 1$
5. $\overline{K}_i^t - 1.96s \leq K_i^t \leq \overline{K}_i^t + 1.96s$

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